Now, lets put m & n Variations together. Each Combination is Called a "Mode". Remember, not all Combinations exist.

(Cross sectional view)

m → angular Variation n → radial Variation

TM modes

m = 0 n = 1



$\bar{E}$ +ve

half wave 1

H → Clkwise

m = 1 n = 2



m = 1 variation

n = 2 variation

half wave 1

half wave 2

m = 0 n = 2



half wave 1

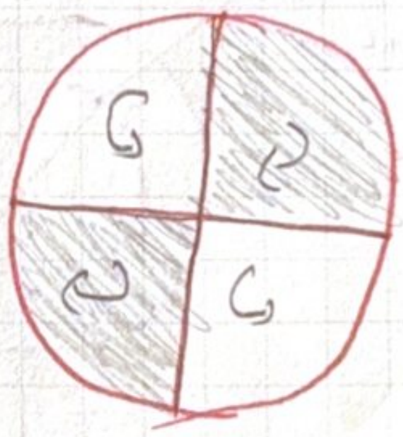
half wave 2

m = 2 n = 0



→ doesn't exist

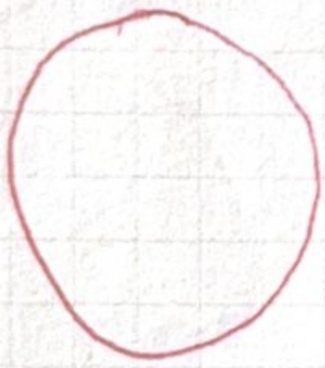
m = 2 n = 1



half wave-1

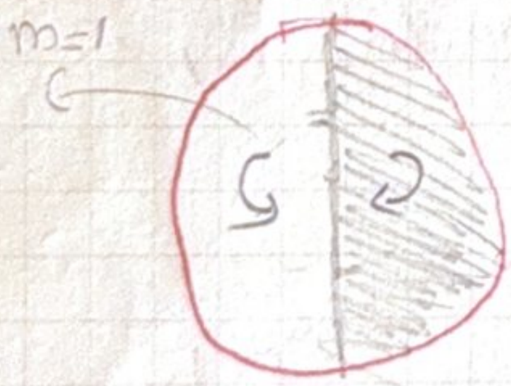
half wave-2

m = 1 n = 0



n = 0 Combination — Doesn't exist

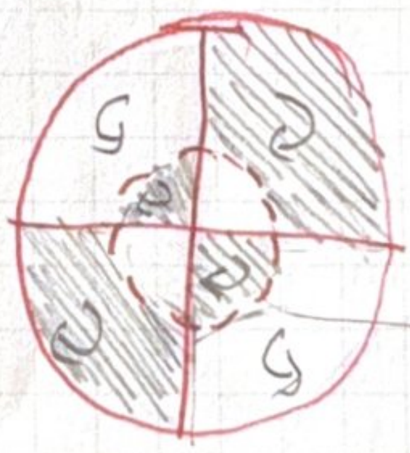
m = 1 n = 1



half wave 1

half wave 2

m = 2 n = 2



half wave-1

half wave-2

n = 2

m = 2

→ is half wave of E_z

→ Another half wave of E_z

2 & G indicate H Circulation

TE Modes

m=0 n=1



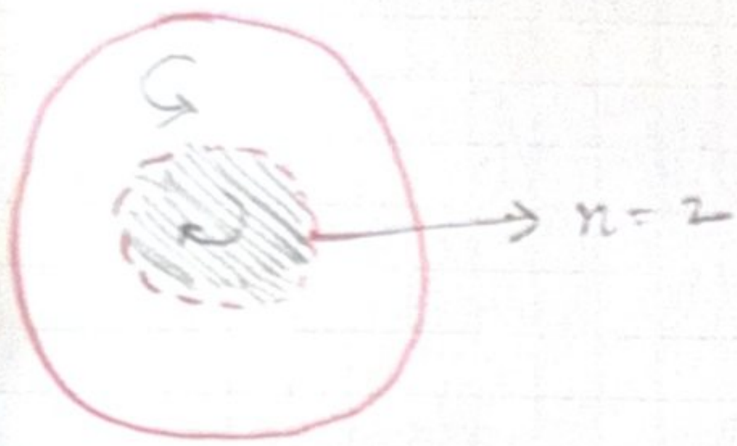
▭ → $\bar{H}$ Circulation in one direction

↺ E_z Circulation

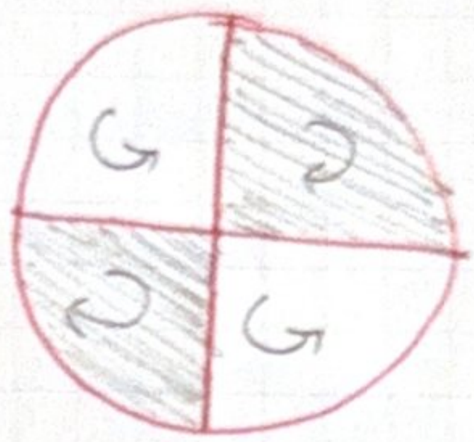
m=1 n=2



m=0 n=2



m=2 n=1



m=1 n=1

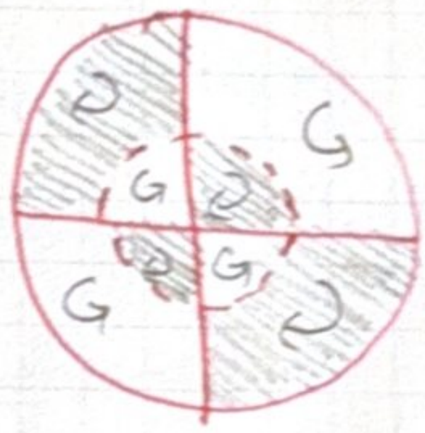


▨ → $\bar{H}$ Circulation in one direction

▭ → $\bar{H}$ Circulation in opp direction

↺ & ↻ $\bar{E}_z$ Circulation

m=2 n=2



→ Here Wall $\frac{\partial H_z}{\partial r} = 0$ For TE Mode ⇒ wall is antinode
↳ neumann condition $J_m' = 0$
↓
max pt
↳ Field → max

For TM mode $E_z = 0$ ⇒ wall is node pt
↳ $J_m = 0$
↓
Field → 0

→ Ring ( ) uses zeros of J_m' for TE & J_m for TM modes
⇒ Ring radius differ

When a wave enters a waveguide, it attempts to excite every single mathematical combination of 'm' & 'n' simultaneously. But this pipe acts as a high pass filter & allows only certain frequencies.

If the wavelength of incoming wave is larger than diameter of the waveguide pipe, it cannot propagate in the pipe & High freq ⇒ Smaller wavelength. So, HPF

Now if the incoming wave freq < specific mode cutoff freq f_c (Varies with m&n) then that mode is Choked out & becomes "Evanacent" wave → decays exponentially within few meters.

only the modes whose calculated cutoff freqs stay lower than incoming freq survive.

Out of all these calculated 'f_c' for combination of m&n, the one 'f_c' combination of specific m&n is termed as dominant mode.

For a W.G, a specific freq window exists just above its cutoff freq of mode where only dominant mode exists.

Now, if the incoming wave freq is high enough or the W.G is wide enough, multiple modes will cross the propagation threshold simultaneously. This causes overmoding.

When multiple modes coexist, they overlap in same space causing Spt Superposition. (Sum of all vectors of all modes in $\vec{E}$ & $\vec{H}$)

This sum produces a pattern based on how much each mode receives energy for Superposition.

If the incoming wave causing Overmode somehow causes the Superposition to mimic a mode (This mode receives more power compared to others) & let say that mode is different from dominant mode. Just because it has more power, the overall mode pattern tends to follow this mode & not dominant mode.

Let's use numbers to understand this long theory

Say radius of W.G pipe is 22 cm

$$f_c \text{ for TE mode} = \frac{C \times P'_{mn}}{2\pi a}$$

$$f_c \text{ for TM mode} = \frac{P_{mn} \times C}{2\pi a \sqrt{\mu\varepsilon}}$$

m, n	P'_{mn}	P_{mn}
0, 1	3.832	2.405
0, 2	7.016	5.520
1, 1	1.841	3.832
1, 2	5.331	7.016
2, 1	3.054	5.136
2 2	6.706	8.417

For above formula, P'_{mn} & P_{mn} decide the lowest value of 'f_c' in TE & TM modes

TE_{11} is the most dominant mode of all next is TM_{01}

TE_{11} $f_c \rightarrow$ 399.6 MHz $\tilde{<}$ 400 MHz

TM_{01} $f_c \rightarrow$ 522.6 MHz

Now if we inject a 600 MHz signal in pipe,

TE_{11} & TM_{01} are allowed to propagate

If our signal injection is such that 85% power couples to TM_{01} mode & 15% to TE_{11} then even though TE_{11} is dominant here, TM_{01} tends to be the overall superposition mode.

[f_c of modes different → phase velocity of modes different ⇒ Superposition will be impacted]

⇓

Causing pattern formation near to the mode the summation mimics

---

Challenge 1 – Math alert!

C-Band Verification Challenge

Design a copper waveguide feed for a C-band test stand.

Inner radius of pipe = 25mm (a)

Operating frequency of RF source = 5 GHz

W.G Interior → filled with Vaccum

Let's Calculate cut off freqs for different modes for this geometry

Let's start with lowest cut off freq & work way up.

From table of P'_{mn} & P_{mn}

TE_{11} → lowest

$$f_c \text{ @ } TE_{11} = \frac{C \times 1.841}{2\pi \times (25 \times 10^{-3})}$$

f_c @ T_{E11} = $3.516\ GHz$

Next T_{M01}

$$f_c \text{ @ } T_{M01} = \frac{2.405}{2\pi \times 10^{-9} \times 25} \times 3\times10^8$$

f_c @ T_{M01} = $4.5932\ GHz$

Next T_{E21}

$$f_c \text{ @ } T_{E21} = \frac{3\times10^8 \times 3.054}{2\pi \times 25\times10^{-3}}$$

f_c @ T_{E21} = $5.83271\ GHz$

Compare this with operating frequency 5 GHz

W.G → HPF

T_{E11} mode — pass
T_{M01} mode — pass
] $f_c < 5\ GHz$ — will propagate

T_{E21} mode - fail — Evanescent death

Let's calculate phase velocities of each mode

Phase Velocity in a W.G $v_p = \frac{c}{\sqrt{1-(f_c/f)^2}}$

$$v_p \rightarrow T_{E11} = \frac{3\times10^8}{\sqrt{1-\left(\frac{3.516}{5}\right)^2}} = 421.94\times10^6\ m/s$$

$$v_p\ TM_{01} \rightarrow \frac{3\times10^8}{\sqrt{1-\left(\frac{4.5932}{5}\right)^2}} = 759.30\times10^6\ m/s$$

TM_{01} wave front (phase front) travels almost twice faster than TE_{11} mode phase front.

[Phase Velocity is how fast a particular pt in a wave Ex: Crest travels along the wave guide geometry.]

This term dictates how signals disperse and distort over long cable/pipe runs.

Let's use an accelerator example to understand this significance

A proton is accleating in W.G pipe. proton has its own ~~Phase~~ Velocity (~~proton acts as wave~~). The $\bar{E}$ field in the accelerator W.G ~~phase~~ pipe has its own phase velocity Let's say its the TM mode one which we Calculated earlier.

If photon travels near to speed of light. It v_p is slightly less more than speed of light & v_p of TM_{01} is 7.593×10^8 m/s. almost twice the speed if v_p (proton) is ~~24~~ 21 $\times10^8$ m/s (say)

By the time photon's Crest matches to Crest of $\bar{E}$ of TM_{01}, ithe trough of $\bar{E}$ appears (as if it decelerates by the time it reaches acclerator (crest pt). It Cannot keep up with the Speed of EM wave & most of the time having net acceleration = 0

To over come this we reduce the v_p of W.G by playing with its geometry (defining) → making a slow wave structure to match this.

Another significance of phase velocity → Spatial beating.

Let's say two modes TM_{01} & TE_{11} coexist in WG due to a wrong feed input. both wave run past each other due to different phase velocities & at some point the crests mated & at some other troughs. They experience superposition at those spatial locations causing spatial beating.

Lets calculate TE & TM mode impedances

For TE mode, Wave impedance > Free space impedance (377Ω)

$$Z_{TE} = \frac{377}{\sqrt{1-(f_c/f)^2}} = \frac{377}{\sqrt{1-\left(\frac{3.516}{5}\right)^2}} = \underline{530.244\ \Omega}$$

For TM mode, wave impedance < Free space impedance (377Ω)

$$Z_{TM} = 377\sqrt{1-(f_c/f)^2} = 377 \times \sqrt{1-\left(\frac{4.5932}{5}\right)^2}$$

$$= \underline{148.951\ \Omega}$$

If we directly inject W.G from standard 50Ω coax, due to these dispersive Impedances (for specific mode), they result in reflections & bad VSWR.

Max power limit of a waveguide — Take it as it is

If $\bar{E}$ exceeds certain value, arcs/sparks arise in a waveguide. — RF Breakdown

In fact ← The Dry air at 1 atm reaches dielectric breakdown when $\bar{E}$ in dry air reaches 3MV/m

To ensure our W.G safe, we must never let $\bar{E}_{max}$ reach 3MV/m if WG is filled with dry air.

Power = $\frac{V^2}{Z}$ we use $\bar{E}$ instead of Voltage

$\therefore P \propto \frac{E^2}{Z_{wave}}$ (why '$\propto$' since $\bar{E}$ & $\bar{V}$ are related by Vector calculus)

$$V = -\int_A^B E \cdot ds$$

$$P = \frac{1}{2Z_w} \iint |E_+|^2 \cdot ds$$

For TE_{11} mode,

→ use this derived formula

$$P_{TE_{11}} = \frac{\pi a^2 E_{max}^2}{2Z_{TE}} \left(1 - \frac{1}{(P'_{11})^2}\right) \left(J_1^2(P'_{11})\right)$$

$P'_{11} = 1.841$

$J_1(P'_{11}) \cong 0.5819$ (From Bessel func table)

$$P_{TE11} = \frac{\pi \times (25\times10^{-3})^2 \times (3\times10^6)^2}{530.244} \left(1 - \frac{1}{(1.841)^2}\right) (0.5819)^2$$

$$\boxed{P_{TE11} = 7.9552\,MW}$$ — Max limit for this geometry, freq of operation

For TM01 mode,

$$P_{TM01} = \frac{\pi a^2 E_{max}^2}{2 Z_{TM}} \left(\left(\frac{k}{k_c}\right)^2 - 1\right) J_1^2(p_{01})$$

$J_1(p_{01}) \propto J_1(2.405) \approx 0.5191$

$p_{01} = 2.405$

$$P_{TM01} = \frac{\pi \times (25\times10^{-3})^2 \times (3\times10^6)^2}{2 \times 148.951} \times \left(\left(\frac{5}{4.5732}\right)^2 - 1\right) \times (0.5191)^2$$

$\boxed{P_{mg} = 2.9567\,MW}$ — Max limit for this mode, geometry & freq of operation

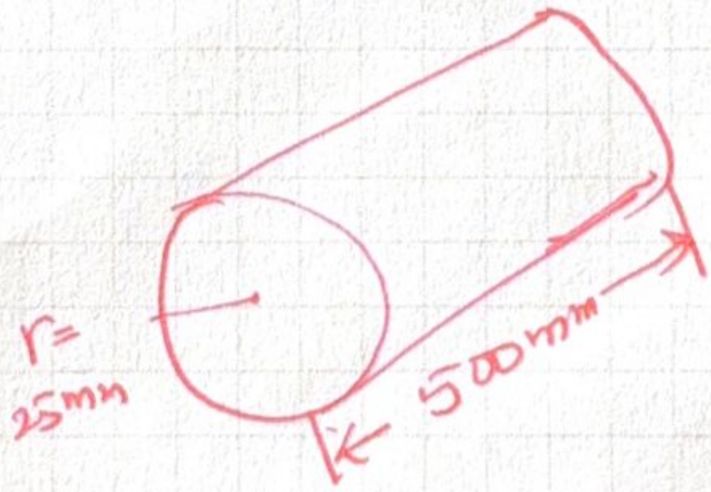
If W.G is filled with Vaccum instead of dry air

$E_{max} = 59.3\,MV/m$ → theoritical limit

$\boxed{P_{TE11} \propto 3.11\,GW}$ & $\boxed{P_{TM01} \simeq 1.15\,GW}$

Lets simulate above challenge in EM tool (COMSOL for now)

Geometry of WG considered for Simulation



f = 5GHz
Material copper
filling - Vaccum

Plan for Simulation – Preprocessing

Geometry → Cylinder rad = 25mm
Length = 500mm → Z-axis
or height

Assuming PEC initially (No lossy material)
Vaccum filled → No another material

→ Study Domain → Electromagnetic wave

Settings → PEC defined only on cylindrical Surface & not on ports (Cross section end)

Port 1 & 2 Numerical type, 1 → excited with 1W & 2 → No excitation

Str Meshing → Basic physics controlled mesh with fine element size

Study → (order to follow) Step 1 Boundary mode analysis

Port 1, Freq 5 GHz
Search for mode shift around
Set 0.711 for TE_{11}

Reason: Eigen Value effective wave number $\sqrt{1-\left(\frac{f_c}{f}\right)^2}$ → 5 GHz
↓
0.711

f_c → From hand calculated Value

↳ This will help to converge faster

For TM_{01} Set → 0.395

Step 2 → Boundary analysis → for Port 2 (same settings)

Step 3 → Freq domain → Set 5 GHz Freq → Run Simulation

Results & EMission — Postprocessing (Very important)

→ Phase Velocity → global Evaluation
↓
C_const / emw.neff_1
↓
c / ~~λ0~~ f Eigen (phase) number

Derived Values

emw.neff_2 → for TM01 (:: it's second cut off number / Eigen number)

→ Wave Impedance → global Evaluation
↓
For TE11 ← Z0 / emw.neff_1 : Z0 / emw.neff_2 ↓ For TM01

→ Power Measurement → Derived Value → Volume mean
↓
Full domain
↓
emw.normE
↓
Enter → Run this Value + copy

↳ Global evaluation → power mean (name)
↓
diel. ~~break~~ in dry air ← (Eeff / Emax Value)² × 1/2
↓
Ans in Megawatts

Another way to Verify Phase Velocity

↓
Datasets → cut line 3D → from 000 to 0 0 500/400
↓ height

→ 1D plot group → line graph → Set the defined cut line

For TE11 ↳ Evaluate Ex or Ey (emw.Ex or Emw.Ey)
↳ measure the wavelength
↳ λ × 5 GHz = Vp

For TM01
Its Ez ⇒ emw.Ez
↓ Measure wavelength
~~λ~~ × 5 GHz = Vp